IYGB GCE

Core Mathematics C3

Advanced

Practice Paper W

Difficulty Rating: 3.94/1.9417

Time: 2 hours

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The curve C has equation

$$y = \frac{kx^2 - a}{kx^2 + a},$$

where k and a are non zero constants.

a) Find a simplified expression for $\frac{dy}{dx}$ in terms of a and k. **(4)**

b) Hence show that C has a single turning point for all values of a and k, and state its coordinates. **(3)**

Question 2

It is given that

$$\frac{2\tan x}{1+\tan^2 x} \equiv \sin 2x .$$

a) Prove the validity of the above trigonometric identity. **(3)**

b) Use part **(a)** to show that

$$\tan 15° = 2 - \sqrt{3} . \quad \textbf{(6)}$$

Question 3

Solve the trigonometric equation

$$7\sin^2 x + \sin x \cos x = 6 , \; 0° \le x \le 360° ,$$

giving the answers to the nearest degree. **(7)**

Question 4

The function f is defined by

$$f(x)=\begin{cases} 4-x, & x\in\mathbb{R},\ x\le 2 \\ 2(x-1)^2, & x\in\mathbb{R},\ x\ge 2 \end{cases}$$

a) Sketch the graph of $f(x)$. **(4)**

b) State the range of $f(x)$. **(1)**

c) Solve the equation

$$f(x)=18.$$ **(4)**

Question 5

$$f(x)=9x^2+6x+2,\ x\in\mathbb{R}.$$

$$g(x)=(x+1)(x+3),\ x\in\mathbb{R}$$

Show clearly that …

a) … $f(x)=|f(x)|$. **(3)**

b) … the equation $g(|x|)=2$ has no solutions. **(3)**

Question 6

It is required to find the approximate coordinates of the points of intersection between the graphs of

$$y_1 = 9 - x^2,\ x \in \mathbb{R} \qquad \text{and} \qquad y_2 = \ln(x-1),\ x \in \mathbb{R},\ x > 1.$$

a) Show that the two graphs intersect at a single point P. **(3)**

b) Explain why the x coordinate of P lies between 2 and 3. **(1)**

The recurrence formula

$$x_{n+1} = \sqrt{9 - \ln(x_n - 1)},$$

starting with a suitable value for x_1, is to be used to find the x coordinate of P.

c) Calculate the x coordinate of P, correct to three decimal places. **(3)**

d) By considering two suitable transformations, determine correct to two decimal places the coordinates of the points of intersection between the graph of

$$y_3 = 3\left[9 - (x+1)^2\right],\ x \in \mathbb{R} \qquad \text{and} \qquad y_4 = 3\ln x,\ x \in \mathbb{R},\ x > 0. \quad \textbf{(5)}$$

Question 7

A curve C has equation

$$x = y\sqrt{9 - 4y^2},\ |y| \leq \frac{3}{2}.$$

a) Show clearly that

$$\frac{dy}{dx} = \frac{\sqrt{9-4y^2}}{9-8y^2}. \quad \textbf{(6)}$$

b) Find the exact coordinates of the two points on C, with infinite gradient. **(4)**

Question 8

A scientist is investigating the population growth of farm mice.

The number of farm mice N, t months since the start of the investigation, is modelled by the equation

$$N = \frac{600}{1+\mathrm{e}^{-0.25t}}, \ t \geq 0 .$$

a) State the number of farm mice at the start of the investigation. **(1)**

b) Calculate the number of months that it will take the population of farm mice to reach 455. **(4)**

c) Show clearly that

$$\frac{dN}{dt} = \frac{1}{4}N - \frac{1}{2400}N^2 . \quad \textbf{(6)}$$

d) Find the value of t when the rate of growth of the population of these farm mice is largest. **(4)**