IYGB GCE

Core Mathematics C3

Advanced

Practice Paper P

Difficulty Rating: 3.4800/1.5873

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 10 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Given that

$$\frac{4x^4+4x^3-23x^2-4}{x^2+x-6} \equiv Ax^2+Bx+C-\frac{C}{x+E},$$

find the value of each of the constants A, B, C, D and E. **(5)**

Question 2

$$\cos^2 x+\sin^2 x \equiv 1.$$

a) Starting with the above identity prove that

$$1+\tan^2 x \equiv \sec^2 x. \quad \textbf{(1)}$$

b) Hence, or otherwise, solve the following trigonometric equation

$$2\tan^2 x+\sec^2 x = 5\sec x, \quad 0° \le x < 360°. \quad \textbf{(6)}$$

Question 3

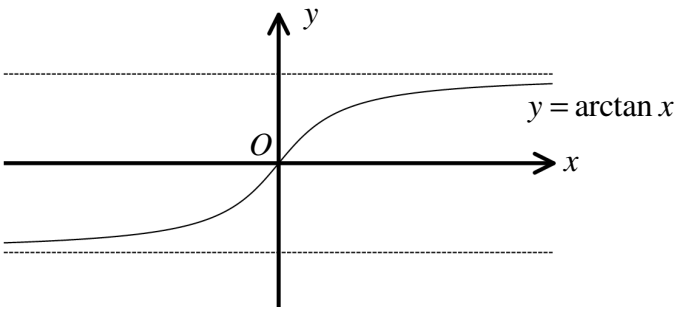
Given that

$$y=\frac{8x^2+8x+3}{(2x+1)^2}, \quad x \neq -\frac{1}{2},$$

show that

$$\frac{dy}{dx}=-\frac{4}{(2x+1)^3}. \quad \textbf{(5)}$$

Question 4



The diagram above shows the graph of

$$y = \arctan x\,.$$

a) Write down the equations of the two horizontal asymptotes. **(1)**

b) Copy the diagram above and use it to show that the equation

$$3x - \arctan x = 1$$

has only one positive real root. **(2)**

c) Show that this root lies between 0.45 and 0.5. **(3)**

d) Use the iterative formula

$$x_{n+1} = \frac{1}{3}\left(1 + \arctan x_n\right),\ x_0 = 0.475$$

to find, to 3 decimal places, the value of x_1, x_2 and x_3. **(2)**

Question 5

The curve has equation

$$y = \mathrm{e}^{2x} - 4\mathrm{e}^{x} - 16x .$$

a) Show that the x coordinates of the stationary points of the curve satisfy the following equation

$$\mathrm{e}^{2x} - 2\mathrm{e}^{x} - 8 = 0 . \qquad (\mathbf{2})$$

b) Hence determine the exact coordinates of the stationary point of the curve, giving the answer in terms of $\ln 2$. $(\mathbf{5})$

Question 6

A curve C is defined by the equation

$$y = -\arcsin(x-1), \ 0 \le x \le 2 .$$

a) Describe the 2 geometric transformations that map the graph of $\arcsin x$ onto the graph of C. $(\mathbf{4})$

b) Sketch the graph of C.

The sketch must include the coordinates of any points where the graph of C meets the coordinate axes and the coordinates of the endpoints of C. $(\mathbf{4})$

Question 7

A curve has equation

$$x = y^2 \ln y, \ y > 0 .$$

Show that an equation of the normal to the curve at the point where $y = \mathrm{e}$ is

$$y + 3\mathrm{e}x = \mathrm{e}\left(3\mathrm{e}^2 + 1\right). \qquad (\mathbf{7})$$

Question 8

$$f(y) = 6 + 3\cos y + 4\sin y, \quad 0 < y < 2\pi .$$

a) Express $3\cos y + 4\sin y$ in the form $a\cos(y - b)$, $a > 0$, $0 < b < \frac{\pi}{2}$. **(4)**

It is further given that for $0 < y < 2\pi$

$$A \leq 2f(2y) \leq B .$$

b) Determine the value of each of the constants A and B. **(2)**

Question 9

The functions f and g are defined as

$$f(x) = |x| - a, \ x \in \mathbb{R},$$

$$g(x) = |2x + 4a|, \ x \in \mathbb{R},$$

where a is a positive constant.

a) Sketch in the same diagram the graph of $f(x)$ and the graph of $g(x)$.

The sketch must include the coordinates of any points where the graphs meet the coordinate axes. **(5)**

b) Find the solutions of the equation

$$|x| - 3 = |2x + 12| . \quad \textbf{(5)}$$

Question 10

The functions f and g satisfy

$$f(x) = 2e^{\frac{1}{2}x},\ x \in \mathbb{R}$$

$$g(x) = \ln 4x \quad x \in \mathbb{R},\ x > \frac{1}{4}.$$

a) Find $fg(x)$ in its simplest form. **(4)**

b) Find the domain and range of $fg(x)$. **(2)**

c) Solve the equation

$$fg(x) = 3x + 1. \quad \textbf{(6)}$$