IYGB GCE

Core Mathematics C3

Advanced

Practice Paper I

Difficulty Rating: 3.6333/1.6901

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 10 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

$$4\arccos x = x + 1,\ x \in \mathbb{R},\ -1 \le x \le 1.$$

The above equation has a single root α.

a) Show that $0.5 < \alpha < 1$. **(3)**

An iterative formula of the form $x_{n+1} = \cos\left(f\left(x_n\right)\right)$ is used to find α.

b) Use this iterative formula, starting with $x_0 = 1$, to find the value of $x_1,\ x_2,\ x_3,\ x_4$ and x_5.

Give the answers correct to 5 decimal places. **(4)**

c) Write down the value of α to an appropriate accuracy. **(1)**

Question 2

$$f\left(x\right) \equiv \frac{4x-1}{2\left(x-1\right)} - \frac{3}{2\left(x-1\right)\left(2x-1\right)} - 2,\ x \in \mathbb{R},\ x \neq 1,\ x \neq \frac{1}{2}.$$

Show clearly that

$$f\left(x\right) \equiv \frac{k}{2x-1},\ x \in \mathbb{R},\ x \neq \frac{1}{2},$$

where k is an integer to be found. **(6)**

Question 3

Find the exact solution of the following equation

$$\mathrm{e}^{x} - \mathrm{e}^{-x} = \frac{3}{2}.$$ **(5)**

Question 4

The function f is defined as

$$f : x \mapsto |2x-5|, \quad x \in \mathbb{R}.$$

a) Sketch the graph of $f(x)$. **(3)**

Mark clearly in the sketch the coordinates of any x or y intercepts.

b) Solve the equation

$$f(x) = x. \quad \textbf{(3)}$$

The function g is defined as

$$g : x \mapsto x^2 - x, \quad x \in \mathbb{R}.$$

c) Solve the equation

$$fg(x) = 7. \quad \textbf{(5)}$$

Question 5

$$f(x) \equiv 27x^3 - 9x - 2, \; x \in \mathbb{R}.$$

a) Show that $(3x+1)$ is a factor of $f(x)$. **(2)**

It is further given that

$$36\cos 2\theta \cos\theta + 9\sin 2\theta \sin\theta = 4.$$

b) Find the possible values of $\cos\theta$. **(8)**

Question 6

The function f is defined as

$$f(x) = \frac{1}{1+\tan x},\ 0 \le x < \frac{\pi}{2}.$$

a) Use differentiation to show that f is a one to one function. **(5)**

b) Find a simplified expression for the inverse of f. **(3)**

c) Determine the range of f. **(2)**

Question 7

A curve C is defined by the equation

$$x = \sec\left(\frac{y}{2}\right),\ 0 \le y < \pi.$$

a) Show clearly that

$$\frac{dy}{dx} = \frac{2}{x\sqrt{x^2-1}}. \quad \textbf{(4)}$$

b) Hence find the exact coordinates of the point on C, where $\frac{dy}{dx} = \sqrt{2}$. **(5)**

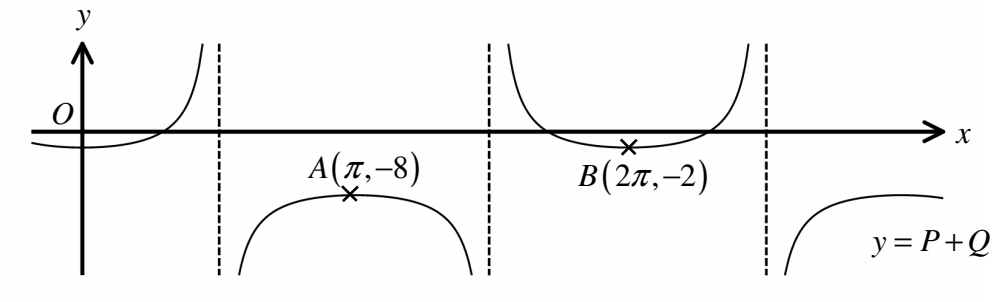
Question 8

By considering a sequence of three transformations, or otherwise, sketch the graph of

$$y = \ln(2-4x),\ x \in \mathbb{R},\ x \le \frac{1}{2}.$$

The sketch must include the coordinates of any points where the graph meets the coordinate axes and the equations of any asymptotes **(4)**

Question 9



The figure above shows part of the curve with equation

$$y = P + Q\sec x,$$

where P and Q are non zero constants.

The curve has turning points at $A(\pi,-8)$ and $B(2\pi,-2)$.

Determine the value of P and the value of Q. **(4)**

Question 10

A curve has equation

$$y = 10\mathrm{e}^{-kx},\ x \in \mathbb{R}$$

where $k = \frac{1}{5}\ln 2$.

Find the value of x that satisfies the equation

$$\frac{dy}{dx} = \ln\left(\frac{\sqrt{2}}{2}\right).$$ **(8)**