IYGB GCE

Core Mathematics C3

Advanced

Practice Paper H

Difficulty Rating: 3.1067/1.3825

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

$$x^3 - x^2 = 6x + 6, \quad x \in \mathbb{R}.$$

a) Show that the above equation has a root α in the interval $(3,4)$. **(2)**

b) Show that the above equation can be written as

$$x = \sqrt{\frac{6x+6}{x-1}}. \quad \textbf{(2)}$$

An iterative formula of the form given in part **(b)**, starting with x_0 is used to find α.

c) Give two different values for x_0 that would not produce an answer for x_1. **(2)**

d) Starting with $x_0 = 3.3$ find the value of x_1, x_2, x_3 and x_4, giving each of the answers correct to 3 decimal places. **(3)**

e) By considering the sign of an appropriate function in a suitable interval, show clearly that $\alpha = 3.33691$, correct to 5 decimal places. **(3)**

Question 2

A cup of tea is cooling down in a room. The temperature T °C of the tea, t minutes after it was made is modelled by the formula

$$T = 20 + 50e^{-\frac{t}{15}}, \ t \geq 0.$$

a) State the temperature of the tea when it was first made. **(1)**

b) Find the temperature of the tea, after 30 minutes. **(1)**

c) Calculate, to the nearest minute, the value of t when the temperature of the tea has reached 35°C. **(4)**

Question 3

The point P, where $x = 2$, lies on the curve with equation

$$y = \frac{1}{6}\left(x^2 + 5\right)^{\frac{3}{2}},\ x \in \mathbb{R}.$$

Find an equation of the tangent to the curve at P. **(6)**

Question 4

$$5\sin 3x\cos x + 5\cos 3x\sin x = 4,\ 0 \le x < \pi.$$

Use a compound angle trigonometric identity to find the solutions of the above trigonometric equation, giving the answers in radians correct to two decimal places. **(6)**

Question 5

$$f(x) \equiv x - \frac{12}{x^2 + 2x - 3} + \frac{3}{x - 1},\ x \in \mathbb{R},\ x \neq -3,\ x \neq 1.$$

Show clearly that

$$f(x) \equiv \frac{x^2 + 3x + 3}{x + 3},\ x \in \mathbb{R},\ x \neq -3,\ x \neq 1.$$ **(7)**

Question 6

The functions f and g are defined as

$$f(x) = \frac{x+6}{x+2}, \quad x \in \mathbb{R},\ x \neq -2$$

$$g(x) = 7 - 2x^2, \quad x \in \mathbb{R}.$$

a) State the range of $g(x)$. **(1)**

b) Find, as a simplified fraction, an expression for $fg(x)$. **(2)**

c) Find, as a simplified fraction, an expression for $f^{-1}(x)$. **(4)**

d) Solve the equation

$$f^{-1}(x) = f(x).$$ **(4)**

Question 7

$$y = \mathrm{e}^{-x} \sin\left(\sqrt{3}x\right),\ x \in \mathbb{R}.$$

Find the exact value of each of the constants R and α so that

$$\frac{dy}{dx} = R\mathrm{e}^{-x} \cos\left(\sqrt{3}x + \alpha\right),$$

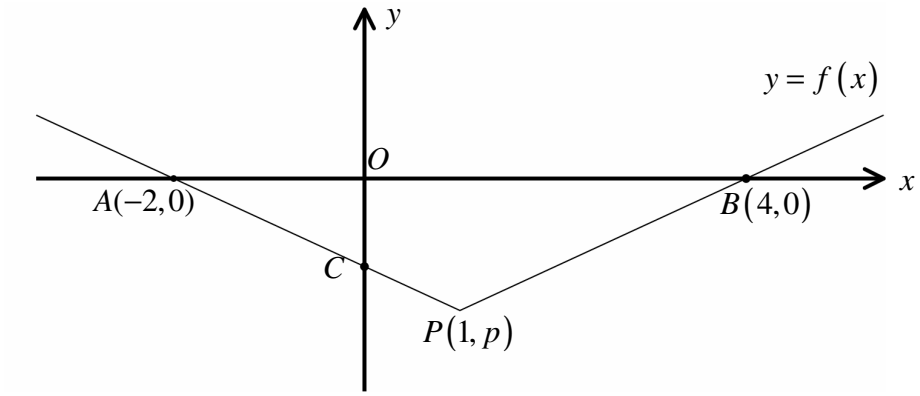
where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(8)**

Question 8

The figure below shows the graph of

$$y = f(x),\ x \in \mathbb{R},$$

which consists of two straight line segments that meet at the point $P(1, p)$.



The points A, B and C are the points where $f(x)$ crosses the coordinate axes.

Sketch, in separate diagrams, the graph of …

a) … $y = f(2x+1)$. **(3)**

b) … $y = f(|x|)$. **(3)**

Each of these sketches must show the coordinates of any intersections with the x axis and the new position of the point P.

It is now given that

$$f(x) = |x-1| - 3,\ x \in \mathbb{R}.$$

c) Find the full coordinates of the points P and C. **(2)**

d) Solve the equation

$$f(x) = 4x.$$ **(5)**

Question 9

If $\cot\theta = 2$, use the trigonometric identity for $\tan 2A$ to show

$$\tan\theta\cot 2\theta\tan 4\theta = -\frac{9}{7}. \qquad (\mathbf{6})$$

You must show detailed workings in this question