IYGB GCE

Core Mathematics C3

Advanced

Practice Paper C

Difficulty Rating: 2.8267/1.26805

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

A curve C has equation

$$y = \sqrt{x^2 + 1}, \ x \in \mathbb{R}.$$

Show that an equation of the normal to C at the point where $x = 1$ is given by

$$y = \sqrt{2}(2 - x). \qquad (7)$$

Question 2

$$e^{-x} + \sqrt{x} = 2$$

a) Show that the above equation has a root α, which lies between 3 and 4. (3)

The recurrence relation

$$x_{n+1} = \left(2 - e^{-x_n}\right)^2$$

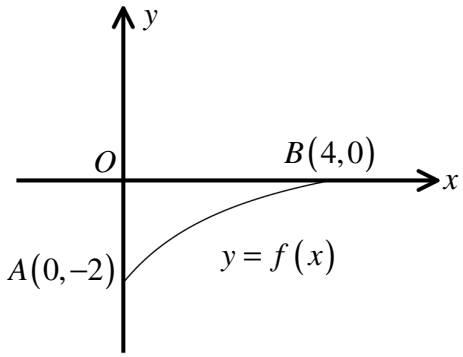
starting with $x_0 = 4$ is to be used to find α.

b) Find, to 3 decimal places, the value of x_1, x_2 and x_3. (3)

c) By considering the sign of an appropriate function $f(x)$ in a suitable interval, show clearly that $\alpha = 3.9211$, correct to 4 decimal places. (2)

Question 3

The figure below shows the graph of the curve with equation $y = f(x)$.



The curve meets the coordinate axes at the points $A(0,-2)$ and $B(4,0)$.

Sketch in a separate diagram the graph of …

a) … $y = f^{-1}(x)$. **(2)**

b) … $y = f(|x|)$. **(2)**

c) … $y = 2f(2x)$. **(2)**

Each sketch must indicate clearly the coordinates of any points where the graph meets the coordinate axes.

Question 4

$$y \equiv 2\sqrt{2}\cos x + 2\sqrt{2}\sin x\,,\ x \in \mathbb{R}\,.$$

a) Express y in the form $R\sin(x+\alpha)$, $R>0$, $0<\alpha<\frac{\pi}{2}$. **(4)**

b) Solve the equation

$$y = 2 \text{ for } 0 < x < 2\pi\,.$$ **(5)**

c) Write down the maximum value of y. **(1)**

d) Find the smallest positive value of x for which this maximum value occurs. **(2)**

Question 5

The curve C has equation

$$y = x\mathrm{e}^{-\frac{1}{2}x^2}\,,\ x \in \mathbb{R}\,.$$

a) Find an expression for $\frac{dy}{dx}$. **(3)**

b) Find the exact coordinates of the turning points of C. **(6)**

Question 6

The function f is defined as

$$f : x \mapsto \frac{1}{x+2} + \frac{2x+11}{2x^2+x-6} \quad x \in \mathbb{R}, \quad x > \frac{3}{2}.$$

a) Show clear that

$$f : x \mapsto \frac{4}{2x-3}, \quad x \in \mathbb{R}, \quad x > \frac{3}{2}. \qquad \textbf{(4)}$$

b) Find an expression for f^{-1}, in its simplest form. **(4)**

c) Find the domain of f^{-1}. **(1)**

The function g is given by

$$g : x \mapsto \ln(x-1), \quad x \in \mathbb{R}, \quad x > 1.$$

d) Show that $x = 1 + \sqrt{e}$ is the solution of the equation

$$fg(x) = -2. \qquad \textbf{(4)}$$

Question 7

Find, in its simplest form, the solution of the following simultaneous equations

$$e^{2x+4} = e\, y$$

$$\ln y = 4x + 6. \qquad \textbf{(6)}$$

Question 8

It is given that

$$\frac{1+\cot^2 x}{\cot x \operatorname{cosec} x} \equiv \sec x .$$

a) Prove the validity of the above trigonometric identity. **(4)**

b) Hence solve the equation

$$\frac{4\left(1+\cot^2 x\right)}{\cot x \operatorname{cosec} x} = \tan^2 x + 5 , \quad 0 \le x < 2\pi ,$$

giving the answers in terms of π. **(7)**

Question 9

Solve the trigonometric equation

$$\pi - 3\arccos\left(x+1\right) = 0 . \qquad \textbf{(3)}$$
